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STEADY-STATE TEMPERATURE FIELD OF A CYLINDRICAL MASS OF RAW MATERIAL WHEN IT IS SELF-HEATED BY AN ELLIPSOIDAL SOURCE

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The axisymmetric distribution of the excess temperature of a fill in the case of the presence of an ellipsoidally shaped thermal source in it has been expanded in a double series. The influence of the site of location and dimensions of the source and of heat-exchange conditions on the values of the steady-state excess temperature has been studied by calculation.

To prevent emergencies (fires, explosions, etc.) caused by the self-heating of raw material in the process of its storage one monitors temperature. It is important to know the temperature fields generated by sources of different shapes. For this purpose we have investigated the distributions of the self-heating temperature in a rectangularly shaped mass [1–3]. Below we consider a cylindrically shaped mass. Such masses occur in practice in the case of storage of hay under a roof and of loose materials in cylindrical silos.

We characterize the dimensions of the mass by radius R_f and height l. The sought distribution function of the excess temperature $T = T(r, z)$ is considered to be dependent just on two variables, i.e., the radial coordinate r and the axial coordinate z. We guide the latter vertically downward, having located its origin on the upper horizontal end of the fill (see Fig. 1). We take an ellipsoid of revolution with semiaxes R_1 and R_3 as the boundary of the internal source of self-heating (region D); this ellipsoid is described, in a cylindrical coordinate system, by the equation

$$\frac{r^2}{R_1^2} + \frac{(z-\zeta)^2}{R_3^2} = 1 .$$

The density of the thermal sources in region D is considered to be constant and equal to q_0. Beyond the region it is equal to zero. The coefficient λ is also considered to be a constant.

With the above simplifications the field of the excess self-heating temperature is described by the Poisson equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = -\frac{1}{\lambda}\begin{cases} q_0 & \text{for} \quad r, z \in D , \\ 0 & \phantom{\text{for}} \quad r, z \notin D . \end{cases} \tag{1}$$

Restricting ourselves to consideration of fills which are stored outdoors (haystacks, strawricks, etc.), on the cylindrical surface $r = R_f$ we take

$$T\,(R_f, z) = 0 . \tag{2}$$

On the fill ends $z = 0$ and $z = 1$, we study three variants of heat-exchange conditions:

(1) the conditions are the same as on the lateral surface, i.e.,

$$T\,(r, 0) = T\,(r, l) = 0 ; \tag{3}$$

(2) the lower end of the mass is ideally heat-insulated; it corresponds to

$$T\,(r, 0) = T_z'\,(r, l) = 0 ; \tag{4}$$

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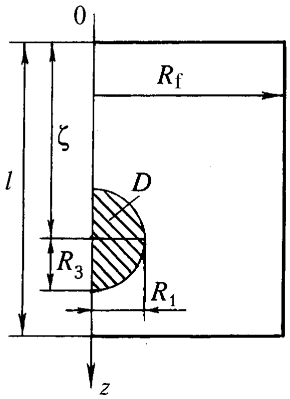

Fig. 1. Computational scheme.

(3) heat insulation of both ends is such that

$$T_z'(r, 0) = T_z'(r, l) = 0 . \quad (5)$$

By expanding the solution of Eq. (1) in a Fourier–Bessel series in the radial coordinate and in a Fourier series in the axial coordinate we find for boundary conditions (2) and (3)

$$T(r, z) = \frac{4q_0}{\lambda l R_f^2} \sum_{m=1}^{\infty} \frac{J_0(\gamma_m r)}{J_1^2(S_m)} \sum_{k=1}^{\infty} \frac{\sin(\mu_k z)\sin(\mu_k \zeta)}{\mu_k^2 + \gamma_m^2} G_{mk} . \quad (6)$$

Here

$$G_{mk} = \frac{2R_1^2 R_3}{\chi_{mk}^2}\left(\frac{\sin \chi_{mk}}{\chi_{mk}} - \cos \chi_{mk}\right); \quad \gamma_m = S_m R_f^{-1} ; \quad \mu_k = k\pi l^{-1} ; \quad \chi_{mk} = (\gamma_m^2 R_1^2 + \mu_k^2 R_3^2)^{1/2} .$$

With boundary conditions (2) and (4) the symbol μ_k in series (6) should be replaced by the symbol $\nu_k = (2k - 1)\cdot\pi(2l)^{-1}$.

In the case of boundary conditions (2) and (5) the solution of the boundary-value problem is the expansion

$$T(r, z) = \frac{4q_0}{\lambda l R_f^2} \sum_{m=1}^{\infty} \frac{J_0(\gamma_m r)}{J_1^2(S_m)} \sum_{k=0}^{\infty} \frac{\cos(\mu_k z)\cos(\mu_k \zeta)}{(\mu_k^2 + \gamma_m^2)(1 + \delta_{k0})} G_{mk} .$$

Let us analyze the results of calculations which have been obtained for the mass of raw material with $R_f = 5$ m and $l = 2R_f$.

The data in Tables 1 and 2 (where the dimensionless values $\overline{T}(r, l/2) = 10^3\lambda T(r, l/2)(q_0 R_f^2)^{-1}$ and $\overline{T}(0, z) = 10^3\lambda T(0, z)(q_0 R_f^2)^{-1}$ are, respectively, indicated) enable one to draw a conclusion on the manner in which the temperature decreases with distance from the center of the source for $R_f R_1^{-1} = 5$.

The accelerated decrease in the temperature over the variable z is attributed to the fact that the ellipsoidal source is flattened in the vertical direction. It has $R_1 = 2R_3$.

The dimensionless values of the temperature $\overline{T}(0, l/2)$ obtained at the centers of the sources of different shapes which have the same volume and hence the same heat-release power are written in Table 3. The quantities R_1 and R_3 were varied so that $R_1^2 R_3 = 1$ m^3 = const.

The calculations show that with the central position of the source (at a large distance from the boundaries of the fill) the largest increase in the temperature in the family of ellipsoidal sources of equal strength is provided by the internal thermal source of a spherical shape ($R_1 = R_3 = 1$ m). Such is not the case for sources located near the boundary (edge) of the fill, which is confirmed by the results of calculations written in Table 4. The table gives the dimensionless values of the temperatures $\overline{T}(0, \zeta) = 10^3\lambda T(0, \zeta)(q_0 R_f^2)^{-1}$ and $\overline{T}(0, l) = 10^3\lambda T(0, l)(q_0 R_f^2)^{-1}$ calculated for a source which

TABLE 1. Values of $\bar{T}(r, l/2)$ for Different r

r, m	0.0	0.3	0.5	1.0	1.5	2.0
$\bar{T}(r, l/2)$	10.864	10.438	9.681	6.134	3.379	2.163

TABLE 2. Values of $\bar{T}$ (0, z) for Different z

z, m	5.0	5.3	5.5	6.0	6.5	7.0
$\bar{T}(0, z)$	10.864	9.915	8.229	4.672	2.962	1.999

TABLE 3. Values of $\bar{T}(0, l/2)$ for Different Shapes of the Sources

R_3, m	$\bar{T}(0, l/2)$	R_3, m	$\bar{T}(0, l/2)$	R_3, m	$\bar{T}(0, l/2)$
0.4	14.734	0.9	17.498	1.2	17.413
0.6	16.569	1.0	17.545	1.4	17.097
0.8	17.348	1.1	17.510	1.6	16.679

TABLE 4. Values of $\bar{T}(0, \zeta)$ and $\bar{T}(0, l)$ for the Bottom Source

R_3, m	$\bar{T}(0, \zeta)$	$\bar{T}(0, l)$	R_3, m	$\bar{T}(0, \zeta)$	$\bar{T}(0, l)$
0.1	14.18	14.36	0.8	23.18	24.08
0.2	19.92	20.52	1.0	22.10	22.06
0.4	23.83	25.24	1.2	21.02	19.96
0.5	24.16	25.71	1.4	20.00	17.98
0.6	24.03	25.53	1.6	19.04	16.19
0.7	23.66	24.93	1.8	18.16	14.61

touches, with its lower point, the lower heat-insulated end of the fill ($\zeta = l - R_3$). In the calculation, we varied the relations of the ellipsoid semiaxes with the strength of the thermal source being constant ($R_1^2 R_3 = 1$ m^3 = const). We took the conditions of ideal heat exchange, i.e., boundary conditions (2) and (4), on the upper end of the fill.

For sources compressed in the vertical direction ($R_3 < 1$) the temperature at the point (0, l) is higher than that at the point (0, ζ). For ellipsoids extended in the vertical direction ($R_3 > 1$) we have the inverse inequality. The maximum increase in the temperature in the family of bottom sources belongs not to a spherical source. The data in Table 4 show that it corresponds to a source which has $R_3 = 0.5$ m and $R_1 \approx 1.4142$ m.

Thus, depending on the location of a source of constant strength we have such relations of its dimensions which give the maximum increase in the temperature of self-heating of raw material.

NOTATION

r and z, radial and axial coordinates; $T(r, z)$, function of the temperature field; R_f and l, radius and height of the cylindrical mass of raw material; λ, thermal conductivity of the raw material; R_1 and R_3, semiaxes of the ellipsoidal source of self-heating; ζ, axial coordinate of the center of the source; $J_0(t)$ and $J_1(t)$, Bessel functions of the first kind of zero and unity orders; S_m, mth positive zero of the function $J_0(S)$; δ_{k0}, Kronecker symbol.

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